IMPACT OF JET

Jet: It's the fluid that comes out of a nozzle.

Impact of jet: It's the force exerted by jet on any surface.

We should understand the following terms while studying impact of jet.

Momentum: It's the quantity of motion a moving body possesses. It is measured as a product of mass and velocity.

Mathematically, momentum = mass X velocity

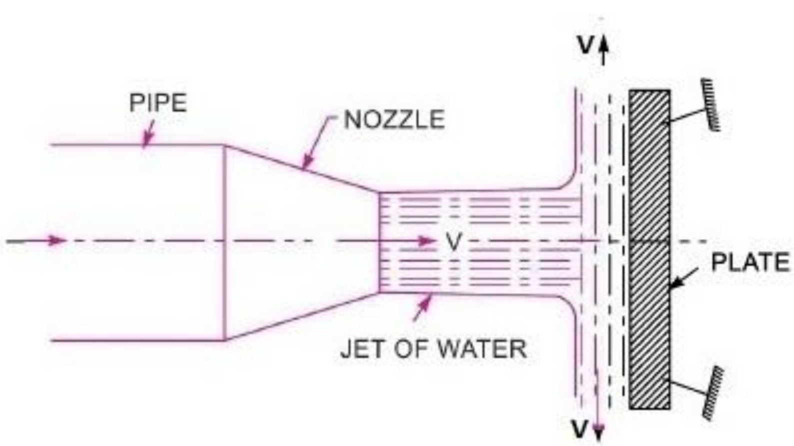
Change of momentum: It's the difference of final momentum and initial momentum.

Mathematically, change of momentum= final momentum - initial momentum

Rate: Any quantity divided by time is called rate of that quantity.

Rate of change of momentum: It's the change of momentum divided by time. This is applicable while calculating force applied on a body by a moving object.

Force exerted by jet on a stationary vertical plate:



The above figure shows a water jet strikes a fixed vertical plate. The jet gets deflected by 90^o after striking the plate.

Let, V= velocity of jet

d= diameter of jet

a= area of cross section of jet= $\frac{\pi}{4}\mathbf{d}^2$

ρ= density of water jet

Force exerted by jet on the plate in the direction of jet,

F_x= Rate of change of momentum in the direction of jet

= (Initial momentum – Final momentum) / Time

$= \frac{(\text{mass x initial velocity})-(\text{mass x final velocity})}{\text{time}}$

$= \frac{\text{mass}}{\text{time}}(\text{initial velocity} - \text{final velocity})$

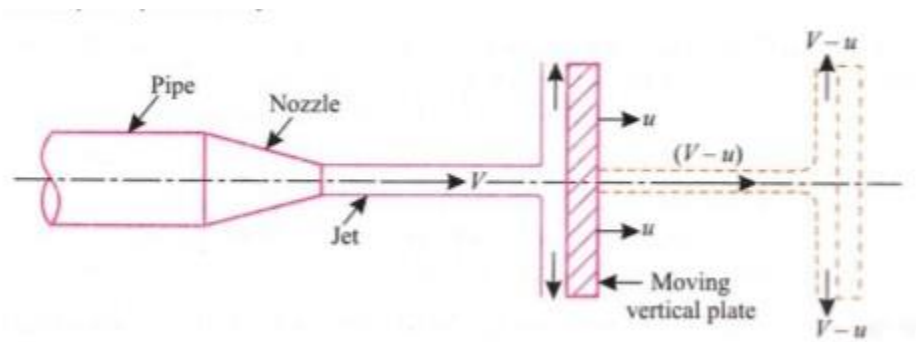
= rate of mass of water striking the jet $(\text{initial velocity} - \text{final velocity})$

$=\rho aV(V-0)$

$= \boldsymbol{\rho a V^2}$

Note: Change of momentum is taken "initial velocity – final velocity" in order to avoid –ve sign of force.

<u>Force exerted by jet on a flat vertical plate moving in the direction of jet:</u>



The above figure shows a water jet strikes a moving vertical plate. The jet gets deflected by 90° after striking the plate.

Let, V= velocity of jet

u= velocity of the flat plate

d= diameter of jet

a= area of cross section of jet= $\frac{\pi}{4}\mathbf{d}^2$

ρ= density of water jet

Rate of mass of water striking the plate

= $\rho a(V-u)$

Force exerted by jet on the moving plate in the direction of jet,

F_x= Rate of mass of water striking the plate

x (initial velocity with which water strikes – final velocity)

= $\rho a(V-u)[(V-u)-0]$

= $\boldsymbol{\rho a(V-u)^2}$

In this case, since the plate is moving, some work is done by the jet on it.

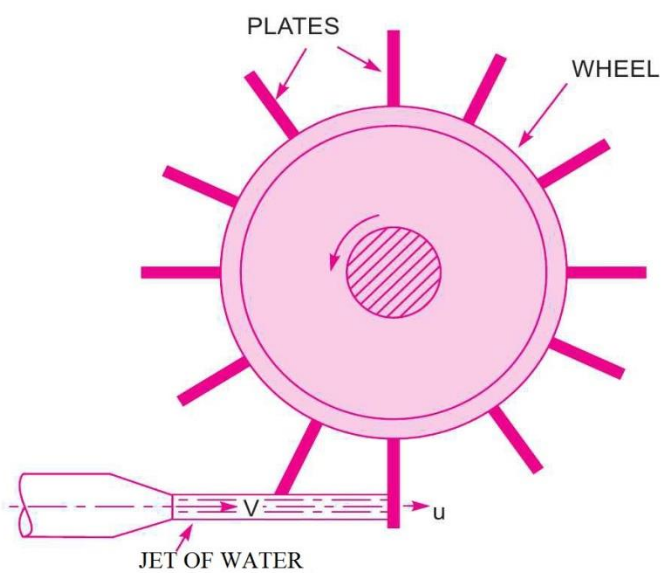
∴ Rate of work done by the jet on the plate

= Force x velocity of plate

= F_x x u

= $\boldsymbol{\rho a(V-u)^2 u}$

Work done on a series of flat vanes:



The previous case of flat plate is of theoretical considerations. The concept of impact of jet is made practically applicable by fixing a series of flat plates (vanes) maintaining equal distance between the plates on the periphery of a rotatable disc as shown in the figure above. The jet exerts force on a plate (vane) fixed on the periphery of the disc. This causes the disc to rotate and brings the next plate on the disc before the jet. This happens for all the plates fixed on the periphery of the disc. As a result, the disc rotates continuously and the power produced due to the rotation of the disc is useful in the field of engineering.

Let, V= velocity of jet
u= velocity of the flat plate
d= diameter of jet
a= area of cross section of jet= $\frac{\pi}{4}\mathbf{d}^2$
ρ= density of water jet

Rate of mass of water striking the series of plates = ρaV **[**Here the jet is always in contact with vane when all the vanes are considered.**]**

The velocity with which the jet strikes the vanes = V – u
It's assumed that the jet moves tangentially to the plate after striking.

Force exerted by the jet in the direction of motion of plate,

F_x= Rate of mass of water(initial velocity – final velocity)
= ρaV[(V-u)-0]
= **ρaV(V-u)**

Rate of work done on the series of flat plates by the jet
= Force x velocity
= F_x x u
= **ρaV(V-u)u**

Rate of kinetic energy of the jet = $\frac{1}{2}\mathrm{mV}^2$
$= \frac{1}{2}\rho aVV^2$
$= \frac{1}{2}\rho aV^3$

$$\therefore \text{Efficiency, } \eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Rate of work done}}{\text{Rate of kinetic energy}} = \frac{\rho aV(V-u)u}{\frac{1}{2}\rho aV^3} = \frac{\mathbf{2u(V-u)}}{\mathbf{V^2}}$$

Condition for maximum efficiency:

For a given jet velocity, the efficiency will be maximum when

$\frac{d\eta}{du} = 0 => \frac{d}{du}\left(\frac{2u(V-u)}{V^2}\right) = 0 => \frac{d}{du}\left(\frac{2uV-2u^2}{V^2}\right) = 0$

$$=> \frac{2V-4u}{V^2} = 0$$

$$=> 2V-4u = 0$$

$$=> 2V = 4u$$

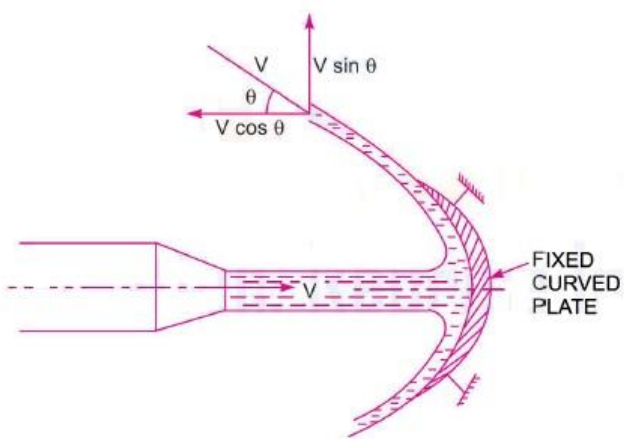
$$=> \mathbf{u} = \frac{V}{2}$$

Maximum efficiency:

$\eta_{max} = \frac{2u(2u-u)}{(2u)^2} = \frac{1}{2} = 0.5 \text{ or } 50\%$

Force exerted by jet on fixed curved plate:

Jet strikes the curved plate at the centre:



A water jet strikes a fixed curved plate at the centre as shown in figure. It is assumed that surface of the plate is smooth. Hence, the jet comes out with same velocity after striking the plate and there is no loss of energy due to impact of jet, in tangential direction of the curved plate.

Let, V= Velocity of the jet at inlet
θ= Angle of jet with the horizontal axis at the outlet.

The velocity of jet at outlet has two components. One is parallel to the jet at inlet and another is perpendicular to the jet at inlet.

Component of jet parallel to the jet at inlet= $-V \cos \theta$

Component of jet perpendicular to the jet at inlet = $V \sin \theta$

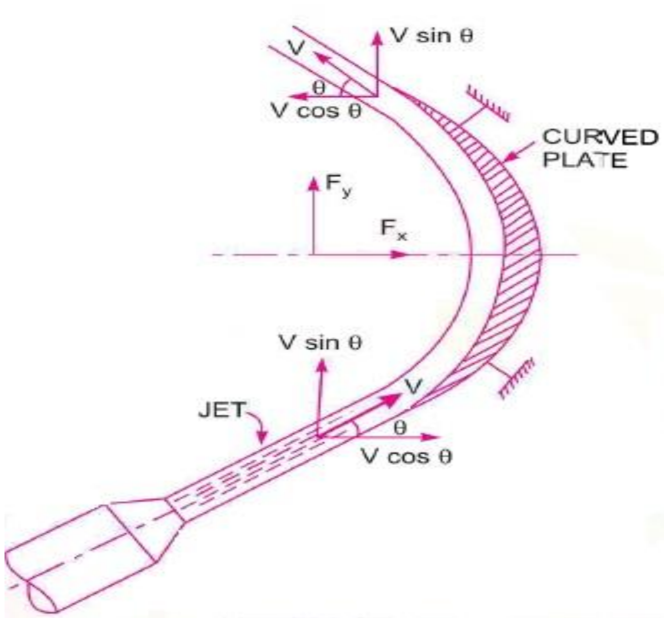
Force exerted by jet in the direction of jet,

F_x = Rate of mass of water (initial velocity-final velocity)

$= \rho a V [V-(-V \cos \theta)]$

$= \boldsymbol{\rho a V^2 (1+ \cos \theta)}$

Jet strikes the curved plate at one end tangentially when the plate is symmetrical:



A jet strikes a fixed curved plate at one end tangentially as shown in figure. The curved plate is symmetrical about horizontal axis. So the angles made by tangents at both the ends will be same.

Let, V= Velocity of jet of water

θ = Angle made by jet with the horizontal axis at the inlet tip of the curved plate.

It's assumed that the plate is smooth and loss of energy due to impact is zero. So the velocity of jet at the outlet tip of plate will be equal to V.

The force exerted by jet in the direction of jet,

F_x= Rate of mass of water (initial velocity - final velocity)

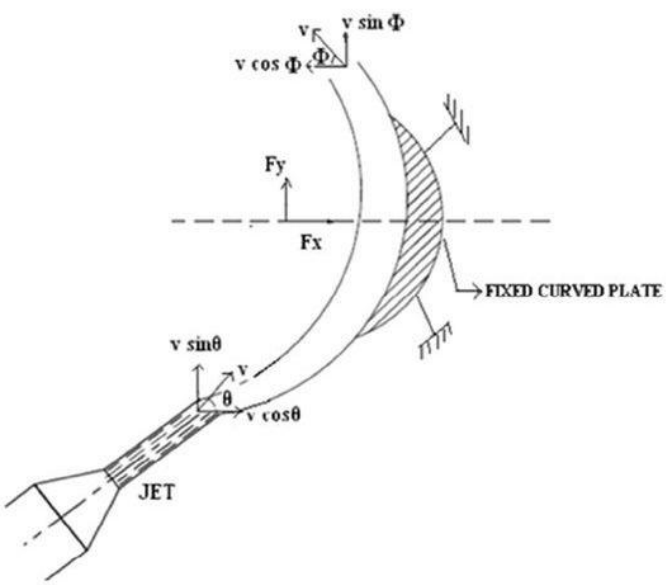
$= \rho aV\ [V \cos\theta - (-V \cos\theta)]$

$= \mathbf{2\ \rho a V^2 \cos\theta}$

F_y = Rate of mass of water (initial velocity - final velocity)

$= \rho aV\ (V \sin\theta - V \sin\theta) = \mathbf{0}$

Jet strikes the curved plate tangentially at one end when the plate is unsymmetrical:



In the figure, the curved plate is unsymmetrical about the horizontal axis. So the angles made by tangents with the horizontal axis at both inlet and outlet tips will be different.

Let, θ = Angle made by tangent at inlet tip with the horizontal axis.

Φ = Angle made by tangent at outlet tip with the horizontal axis.

Force exerted by jet of water in the horizontal direction,

F_x= Rate of mass of water (initial velocity - final velocity)

$= \rho aV [V \cos\theta - (-V \cos\Phi)]$

$= \boldsymbol{\rho a V^2 (\cos\theta + \cos\Phi)}$

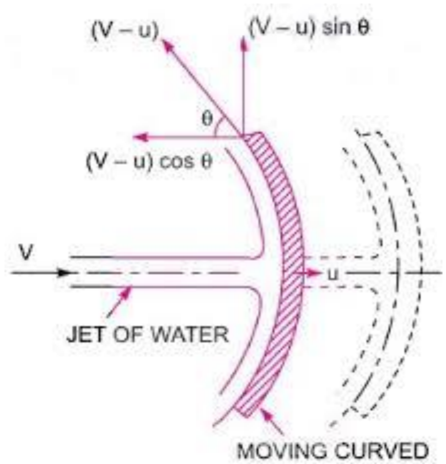
Force exerted by jet of water in the vertical direction,

F_y= Rate of mass of water (initial velocity - final velocity)

$= \rho aV (V \sin\theta - V \sin\Phi)$

$= \boldsymbol{\rho a V^2 (\sin\theta - \sin\Phi)}$

Impact of jet on symmetrical moving curved plate:



A jet of water strikes at the centre of a moving curved plate. The curved plate is symmetrical about the horizontal axis.

Let, V= Velocity of jet

a= Cross sectional area of the jet

u= Velocity of plate in the direction of the jet.

V_r= Relative velocity = V – u

θ = Angle made by tangents with the horizontal axis at both the tips.

It's assumed that the loss of energy due to impact of jet is zero and the surface of the curved plate is smooth. So the velocity with which the jet strikes the plate is equal to velocity with which it leaves the plate.

Force exerted by jet on the curved plate in the horizontal direction,

F_x= Rate of mass of water (initial velocity - final velocity)

$= \rho a V_r [V_r - (- V_r \cos\theta)]$

$= \rho a V_r (V_r + V_r \cos\theta)$

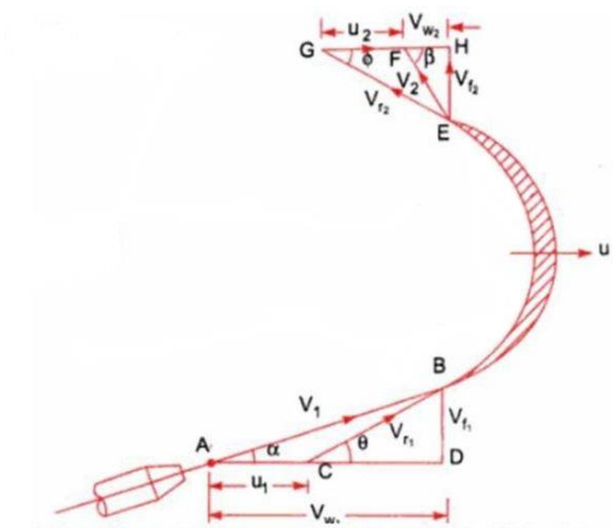
$= \boldsymbol{\rho a V_r^2 (1 + \cos\theta)}$

Rate of work done by the jet on the plate

$= F_x \times u$

$= \boldsymbol{\rho a V_r^2 (1 + \cos\theta) u}$

Force exerted by jet on unsymmetrical moving curved plate when the jet strikes tangentially at one of the tips:



The above figure shows a water jet strikes tangentially an unsymmetrical curved plate at one of the tips. Since the jet strikes the plate tangentially, the loss of energy due to impact of jet will be zero. In this case, since the plate is moving, the velocity with which the jet strikes is equal to relative velocity of jet with respect to the plate. Also the jet leaves the plate tangentially at the outlet.

Let, V_1= Velocity of jet at inlet

u_1= Velocity of plate at inlet

V_{r1}= Relative velocity of jet w.r.t plate at inlet

α= Angle of jet with the direction of motion of plate
= Guide blade angle.

θ = Angle made by relative velocity with the direction of motion of plate at inlet.
= Vane angle at inlet.

V_{w1}= Component of V_1 in the direction of motion of the plate.
= velocity of whirl at inlet.

V_{f1}= Component of V_1 in the direction perpendicular to motion of the plate.
= Velocity of flow at inlet.

V_2, u_2, V_{r2},V_{w2}, V_{f2} = Symbols of corresponding parameters at the outlet of the plate.

β = Angle made by V_2 with the motion of the plate.

ϕ = Angle made by relative velocity with the direction of motion of the plate at the outlet.

a= Cross sectional area of the jet

It's assumed that the plate is smooth. The velocity of plate both at inlet and outlet is same.

i.e. $u_1=u_2$

So, $V_{r1}= V_{r2}$

Rate of mass of water = $\rho a V_{r1}$

Force exerted by jet in the direction of motion of plate,

F_x= Rate of mass of water (initial velocity - final velocity)

$$= \rho a V_{r1}[V_{r1}\cos\theta - (-V_{r2}\cos\phi)]$$
$$= \rho a V_{r1}\{(V_{w1}- u_1) - [-(u_2+V_{w2})]\}$$
$$= \rho a V_{r1}[(V_{w1}- u_1) + (u_2+V_{w2})]$$
$$= \rho a V_{r1}(V_{w1}- u_1 + u_2+V_{w2})$$
$$= \boldsymbol{\rho a V_{r1} (V_{w1} + V_{w2})}$$

Above equation is true only when $\beta < 90^o$.

When $\beta = 90^o$, $V_{w2} = 0$

Therefore, $\mathbf{F_x = \rho a V_{r1} V_{w1}}$

When $\beta > 90^o$, $\mathbf{F_x = \rho a V_{r1} (V_{w1} - V_{w2})}$

Thus the general equation of force exerted by jet striking tangentially a moving unsymmetrical curved plate,

$\mathbf{F_x = \rho a V_{r1} (V_{w1} \pm V_{w2})}$

Rate of work done = F_x x u

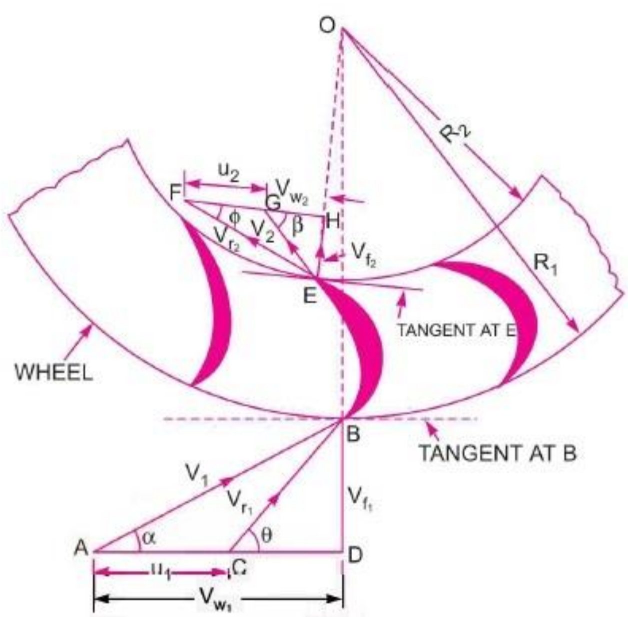
$= \mathbf{\rho a V_{r1} (V_{w1} \pm V_{w2})\, u}$

Efficiency of jet:

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Rate of work done}}{\text{Rate of kinetic energy}} = \frac{\rho a Vr1\ (Vw1 \pm Vw2)\ u}{\frac{1}{2}\rho a V1^3}$$

$$= \frac{\mathbf{2Vr1\ u(Vw1 \pm Vw2)}}{\mathbf{V1^3}}$$

<u>Force exerted on a series of radial curved vanes:</u>



Above figure shows a series of radial curved vanes mounted on a wheel. The jet of water strikes the vanes and the wheel starts rotating at a constant angular speed.

R_1= Radius of wheel at inlet of the vane.

R_2= Radius of wheel at outlet of the vane.

ω = Angular speed of the wheel.

u_1= Velocity of vane at inlet = ωR_1

u_2= Velocity of vane at outlet = ωR_2

a = Cross sectional area of jet

V_1= Velocity of jet at inlet

Rate of mass of water striking the series of vanes = $\rho a V_1$

Rate of momentum of water striking the vanes in the direction tangential to wheel at inlet = Rate of mass of water x Component of V_1 in tangential direction

$= \rho a V_1 \times V_{w1}$

Rate of momentum of water striking the vanes in the direction tangential to wheel at outlet = Rate of mass of water x Component of V_2 in tangential direction

$= - \rho a V_1 \times V_{w2}$

Rate of angular momentum at inlet

= Rate of momentum at inlet x Radius of wheel at inlet

$= \rho a V_1 \times V_{w1} \times R_1$

Rate of angular momentum at outlet

= Rate of momentum at outlet x Radius of wheel at outlet

$= - \rho a V_1 \times V_{w2} \times R_2$

Torque exerted by water on the wheel,

T= Rate of change of angular momentum

= Rate of angular momentum at inlet - Rate of angular momentum at outlet

$= [(\rho a V_1 \times V_{w1} \times R_1) - (- \rho a V_1 \times V_{w2} \times R_2)]$

$= \boldsymbol{\rho a V_1(V_{w1} R_1 + V_{w2} R_2)}$

Rate of work done = Torque x angular velocity

$= \rho a V_1(V_{w1} R_1 + V_{w2} R_2) \times \omega$

$= \rho a V_1(V_{w1} R_1 \omega + V_{w2} R_2 \omega)$

$= \boldsymbol{\rho a V_1(V_{w1} u_1 + V_{w2} u_2)}$

If $\beta > 90^o$, Rate of work done = $\boldsymbol{\rho a V_1 (V_{w1}\, u_1 - V_{w2}\, u_2)}$
General equation for rate of work done = $\boldsymbol{\rho a V_1 (V_{w1}\, u_1 \pm V_{w2}\, u_2)}$

Efficiency:

$$\eta = \frac{\text{Output}}{\text{Input}} = \frac{\text{Rate of work done}}{\text{Rate of kinetic energy}}$$

$$= \frac{\boldsymbol{\rho a V1 (Vw1\ u1 \pm Vw2\ u2)}}{\boldsymbol{\frac{1}{2} \rho a V1^3}}$$

$$= \frac{2\boldsymbol{(Vw1\ u1 \pm Vw2\ u2)}}{\boldsymbol{V1^2}}$$